On the Fate of Close-in Extrasolar Planets

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ABSTRACT

It has been shown that there is a possible mass-period correlation for extrasolar planets from the current observational data and this correlation is, in fact, related to the absence of massive close-in planets, which are strongly influenced by the tidal interaction with the central star. We confirm that the model in Pätzold & Rauer (2002) is a good approximation for the explanation of the absence of massive close-in planets. We thus further determine the minimum possible semimajor axis for these planets to be detected during their lifetime and also study their migration time scale at different semimajor axes by the calculations of tidal interaction. We conclude that the mass-period correlation at the time when these planets were just formed was less tight than it is now observed if these orbital migrations are taken into account.

Subject headings: celestial mechanics – planetary systems – solar system: formation – solar system: general – stellar dynamics

1. Introduction

The number of discovered extrasolar planets is increasing quickly during recent years. According to the Extrasolar Planets Catalog maintained by Jean Schneider (http://cfa-www.harvard.edu/planets/catalog.html), in May 2002, there are about 77 extrasolar planets around 69 main sequence stars. These planets with mass range from 0.16 to 17 Jupiter masses (M_J) have semimajor axes from 0.04 AU to 4.5 AU and also a wide range of eccentricities. Interestingly, there is a planet moving on an extremely elongated orbit ($e = 0.927$) around the solar-type star HD 80606 (Naef et al. 2001). These exciting discoveries provide great opportunities to understand the formation and evolution of planetary systems.

For example, Jiang & Ip (2001) showed that the interaction with disc is important to explain the original orbital elements during the planetary formation. Yeh & Jiang (2001) analytically showed that the scattered planets should in general move on an eccentric orbit and thus the orbital circularization must be important for scattered planets if they are now moving on nearly circular orbits (See Jiang & Yeh 2002a, Jiang & Yeh 2002b for the following up).

In addition to the dynamical studies, Tabachnik & Tremaine (2002) used the maximum likelihood method to estimate the mass and period distributions of extrasolar planets and found there is a mass-period correlation, but they attributed their finding to the observational selection effect.

However, Zucker & Mazeh (2002) claimed that this mass-period correlation cannot be completely explained by the observational selection effect. They did some Monte Carlo simulations and show the real dependency between the mass and period of extrasolar planets. This mass-period correlation gives the paucity of massive close-in planets. Since they are supposed to be the easiest to detect, Zucker & Mazeh (2002) said this paucity was unlikely to be the result of any selection effect.

Pätzold & Rauer (2002) have reported the possible explanation about the absence of massive close-in planets by tidal interaction. They defined "critical mass" to be the maximum mass that the planet can have and survive under the tidal interaction from the central star for a given particular semimajor axis. They determined the critical mass as function of semimajor axis for some assumed stellar dissipation factors and the ages of the planetary systems. Their results showed that most planetary systems are located at the permitted region of the "critical mass-semimajor axis" plot (their Figure 3) except the τ Boo system, which needs more careful treatment for the assumed parameter values.

However, if these planets could be formed a bit farther from the central star initially, they should still survive under the tidal interaction and thus might be detected during the inward migration. One should keep in mind that the location where the planets are detected are not where they are formed. The planets from farther place could migrate inward to the region closer to the central star and probably have chances to be detected by us.

To further investigate this problem, we carefully study the planetary migration due to tidal interaction. We try to include the effect of orbital eccentricity at the beginning and we confirm that that the model used in Pätzold & Rauer (2002) is a good approximation. We thus use the similar model in Pätzold & Rauer (2002) for the rest calculations. We describe our basic models for tidal interaction in Section 2 and the results will be in Section 3. We provide concluding remarks in Section 4.

2. The Models for Tidal Interaction

A tide is raised on the central star by the close-in planet because the force experienced by the side of the central star facing the planet is stronger than that experienced by the far side of the central star. We consider below the models for planets on both circular and eccentric orbits.

2.1. Circular Orbits

If the close-in planet is moving on a circular, equatorial orbit, according to the tidal potential theory, this planet would change its orbit following below formula:

$$\frac{da}{dt} = \text{sign}(\Omega - n)\frac{3k}{Q}\frac{m}{M}(\frac{R}{a})^5 na, \tag{1}$$

where a is the semimajor axis, t is the time, Ω is the rotating angular speed of the central star, k is the stellar Love number, Q is the tidal dissipation function, m is the planetary mass, M is the mass of the central star, R is the central star's radius and n is the orbital mean motion which is determined by

$$n = \sqrt{\frac{G(M+m)}{a^3}}. \tag{2}$$

We set $k = 0.2$ (Murray & Dermott 1999) and take $Q = 3.0 \times 10^5$ (the average value in Pätzold & Rauer 2002).

The above formula provides a good simple tool to study the tidal orbital decay for close-in planets. However, in fact, most discovered planets have certain amount of orbital eccentricities. Some of these eccentricities are even very big. We plan to include the effect of eccentricity into the calculations by the following equations.

2.2. Eccentric Orbits

We know that the angular momentum is related to orbital eccentricity e. Thus, the evolution of semimajor axis a due to tidal interaction should depend on eccentricity e

because the tidal torque change the orbital angular momentum of planets.

The mechanical energy decreasing rate dE/dt due to tidal interaction is

$$\frac{dE}{dt} = \Gamma(\Omega - \frac{d\theta}{dt}), \tag{3}$$

where Γ is the magnitude of the torque, Ω is the spin angular speed of the central star and $d\theta/dt$ is the orbital angular speed of the planet at particular time.

Γ can be approximated by:

$$\Gamma = \frac{3}{2}k\frac{Gm^2}{a^6}R^5\frac{1}{Q}, \tag{4}$$

these parameters are defined in last sub-section.

The orbital angular speed of the planet can be expressed as

$$\frac{d\theta}{dt} = \frac{h}{r^2}, \tag{5}$$

where $h = \sqrt{G(M+m)a(1-e^2)}$ and r is approximated as $r = a(1 - e\cos nt)$.

Therefore,

$$\frac{dE}{dt} = \frac{3}{2}k\frac{Gm^2}{a^6}R^5\frac{1}{Q}[\Omega - \frac{\sqrt{G(M+m)a(1-e^2)}}{a^2(1-e\cos nt)^2}] \tag{6}$$

On the other hand, the mechanical energy of the system can be expressed as

$$E = \frac{1}{2}I\Omega^2 - G\frac{Mm}{2a} \tag{7}$$

and

$$\frac{dE}{dt} = I\Omega\frac{d\Omega}{dt} + G\frac{Mm}{2a^2}\frac{da}{dt} \tag{8}$$

By Kepler's third law,

$$G(M+m) = n^2a^3, \tag{9}$$

we have

$$\frac{dE}{dt} = I\Omega\frac{d\Omega}{dt} + \frac{Mm}{2(M+m)}n^2a\frac{da}{dt}. \tag{10}$$

Further, the angular momentum of the system is

$$L = I\Omega + \frac{Mm}{M+m}a^2n(1-e^2)^{1/2}, \tag{11}$$

where I is the moment of inertia of the central star, e is the orbital eccentricity and we have ignore the contribution from the spin of the planet.

By the conservation of angular momentum, $dL/dt = 0$, we have

$$I\frac{d\Omega}{dt} = -\frac{1}{2}\frac{Mm}{M+m}na\frac{da}{dt}\sqrt{1-e^2} + \frac{Mm}{M+m}na^2\frac{ede/dt}{\sqrt{1-e^2}}. \tag{12}$$

In general, both terms on the right hand side of Equation (12) should be considered. The second term divided by the first term would be

$$e^2(1-e^2)[\frac{63}{6}\frac{Q}{k\mu_pQ_p}(\frac{M}{m})^2(\frac{R_p}{R})^5], \tag{13}$$

where Equation (4.198) in Murray & Dermott (1999) has been used to estimate the value of de/dt and we use μ_p, Q_p and R_p etc. to replace the corresponding parameters $\tilde{\mu_s}$, Q_s and C_s etc. of Equation (4.198) in Murray & Dermott (1999). If we use the Jupiter as an example, this ratio would be about 1 when $e = 0.1$ and $Q/(k\mu_pQ_p) = 1$.

We plan to consider the simple case when $e^2Q/(k\mu_pQ_p)$ is small enough and the second term can be ignored. We will leave more general case in which both orbital migration and circularization need to be included to the future work.

Thus,

$$\frac{dE}{dt} = \frac{1}{2}(n - \Omega\sqrt{1-e^2})\frac{Mm}{M+m}na\frac{da}{dt} \tag{14}$$

From Equation (14) and Equation (6), we have

$$\frac{da}{dt} = 3k\frac{Gm^2}{a^7}\frac{R^5}{Qn}\frac{M+m}{Mm}(n-\Omega\sqrt{1-e^2})^{-1}[\Omega - \frac{\sqrt{G(M+m)a(1-e^2)}}{a^2(1-e\cos nt)^2}], \tag{15}$$

where Ω is related to a by Equation (11).

Given an assumed initial angular momentum L etc., a can be solved numerically by Equation (15).

3. Results

By the equations in last section, we can study the inward migration of planets due to tidal interaction. We place the planet at different initial semimajor axis as different case: 0.02, 0.03, 0.04, 0.05 and also 0.06 AU. Figure 1 are the plots of semimajor axis as function of time for these different initial semimajor axes when we set the planetary mass to be particular value. Thus, there are five curves on each panel of Figure 1. Figure 1(a)-(d) are the results when the planetary masses are assumed to be $5M_J$, $2M_J$, M_J, $0.5M_J$ individually. Since 2 Gyrs is about the age of τ Boo system and thus we regard 2 Gyrs as the typical age of extrasolar planetary systems. Those planets who can survive for 2 Gyrs under the tidal interaction are possible to be detected.

All the curves are the results when we assume the planets move on circular orbits and the triangle points are the results when the planets move on eccentric orbits (assume $e = 0.5$ and $e^2Q/(k\mu_pQ_p)$ is small enough). In general, the results of eccentric orbits are quite similar to the results of circular orbits and the ignorance of eccentricity will not affect the determination of planet surviving time scale etc. This confirms that the equations used in Pätzold and Rauer (2002) are good approximations and we thus use the model of circular orbits for all the rest calculations.

Figure 1(a)-(d) show that when initial semimajor axis $a_i = 0.06$ AU the planet would only have tiny migration during 2 Gyrs. The planet can easily survive under the tidal interaction. If initial semimajor axis $a_i = 0.05$ AU, the orbital semimajor axis decays a bit more. If the initial semimajor axis $a_i = 0.04$ AU, the planet fall into the central star when t is about 1 Gyrs for the case of $5M_J$ but still survive for all other cases. When initial semimajor axis $a_i = 0.03$ AU, the planet falls into the central star within 1.5 Gyrs. If initial semimajor axis $a_i = 0.02$ AU, the planet almost approaches to the central star immediately.

The detection probability for particular range of semimajor axis depends on how much time the planet can survive around that range. We plot the time the planet should spend from one semimajor axis a_j to another semimajor axis a_{j+1} (we assume $a_j > a_{j+1}$) during the orbital decay in Figure 2. There are two sets of a_j: one makes $\delta a \equiv a_j - a_{j+1} = 0.005$ AU (dotted lines), another set $\delta a \equiv a_j - a_{j+1} = 0.0025$ AU (solid lines). Figure 2(a)-(d) are the results when we set the planetary mass to be $5M_J$, $2M_J$, M_J, $0.5M_J$ individually.

In Figure 2(a)-(b), the planet spends more than 1 Gyrs to stay around 0.05 AU and thus the planet is likely to survive during 2 Gyrs. However, the planet only stays around 0.04 AU for about 0.5 Gyrs and around 0.03 AU for about 0.2 Gyrs only. These time scales are considerablely smaller than the age of the planetary system and thus the planet initially formed around these locations are very unlikely to be observed.

When the planetary mass is smaller as in Figure (c)-(d), the planet can survive for much longer (more than 1.5 Gyrs) around 0.04 AU and still only stays around 0.03 AU for order of 0.5 Gyrs. This implies that the probability that the planet is detected to be around 0.03 AU is very small.

Figure 3 are the $\ln(a/\mathrm{AU}) - \ln(M/M_J)$ plots for all discovered extrasolar planets, where M is the planetary mass and a is the semimajor axis. The data for these planets are from Extrasolar Planets Catalog (http://cfa-www.harvard.edu/planets/catalog.html) in May 2002. In Figure 3(a), we take a to be the values of current semimajor axes of these discovered extrasolar planets. However, in Figure 3(b)-(d), we take a to be the planetary semimajor axes backward in time for 2, 6, 12 Gyrs individually. The values of a backward in time can be obtained by Equation (1).

In Figure 3(b)-(d), we found that most planets do not move on the $\ln(a/\mathrm{AU}) - \ln(M/M_J)$ plane but some of them do move a lot when they are backward in time.

It is quite obvious that the planets line up on the left side of the plots in Figure 3(b)-(d) and the position of this line hardly moves from Figure 3(b) to 3(d). This line can be approximated by

$$\ln(M/M_J) = \frac{1}{5}[28\ \ln(a/\mathrm{AU}) + 62]. \tag{16}$$

We can also see that those planets do not move much are all on the right side of this line. This line can thus be regarded as the "critical line": all planets on the right side of this line would not migrate much during their lifetime but all the planets standing on this critical line of Figure 3(b)-(d) would move to the left-up corner of Figure 3(a) after 2, 6 or 12 Gyrs and finally all the planets on the left side of this line would migrate inward quickly to approach the central star and thus cannot be detected.

4. Concluding Remarks

As dynamical friction successfully explained the orbit of Sagittarius dwarf galaxy (Jiang & Binney 2001), the tidal interaction can indeed explain the current observed mass-period correlation reported by Zucker & Mazeh (2002). The results in Figure 1 give us the full picture of inward migration due to tidal interaction. We found that 0.03 AU seems to be the critical semimajor axis for the planet with mass of order of τ Boo system to survive in 2 Gyrs. This is consistent with the current observational results that the smallest semimajor axis of discovered planet is about 0.04 AU.

On the other hand, we can also check this minimum possible semimajor axis from another point of view. In Figure 2, the time scale for a planet can survive is smaller if the planet is closer to the central star initially and the time a planet can stay around 0.03 AU is considerablely much less than 2 Gyrs, which was regarded as the typical age of these planetary systems. Because time scale is too short, the probability to detect the planet is very small.

Moreover, we interestingly discover the observational "critical line" on $\ln(a/\mathrm{AU}) - \ln(M/M_J)$ plane. All the planets on the left side of this line would migrate inward quickly to approach the central star and thus cannot be detected.

Therefore, the initial configuration on $\ln(a/\mathrm{AU}) - \ln(M/M_J)$ plane might be composed of all the points on Figure 3(b) plus those points which might have been on the left side of the "critical line" about 2 Gyrs ago but disappear in Figure 3(a) because these planets already fall into the central star. From this point of view, even there is correlation between mass and period for current discovered planets as claimed by Zucker & Mazeh (2002), this correlation could be weaker or less obvious at the time when these planets were just formed since we can add arbitrary number of "possible" planets on the left side of our observational "critical line" if there is no difficulty to form planets there in theory. This tells us that we should be careful when we try to link the mass-period correlation to the theory of planetary formation.

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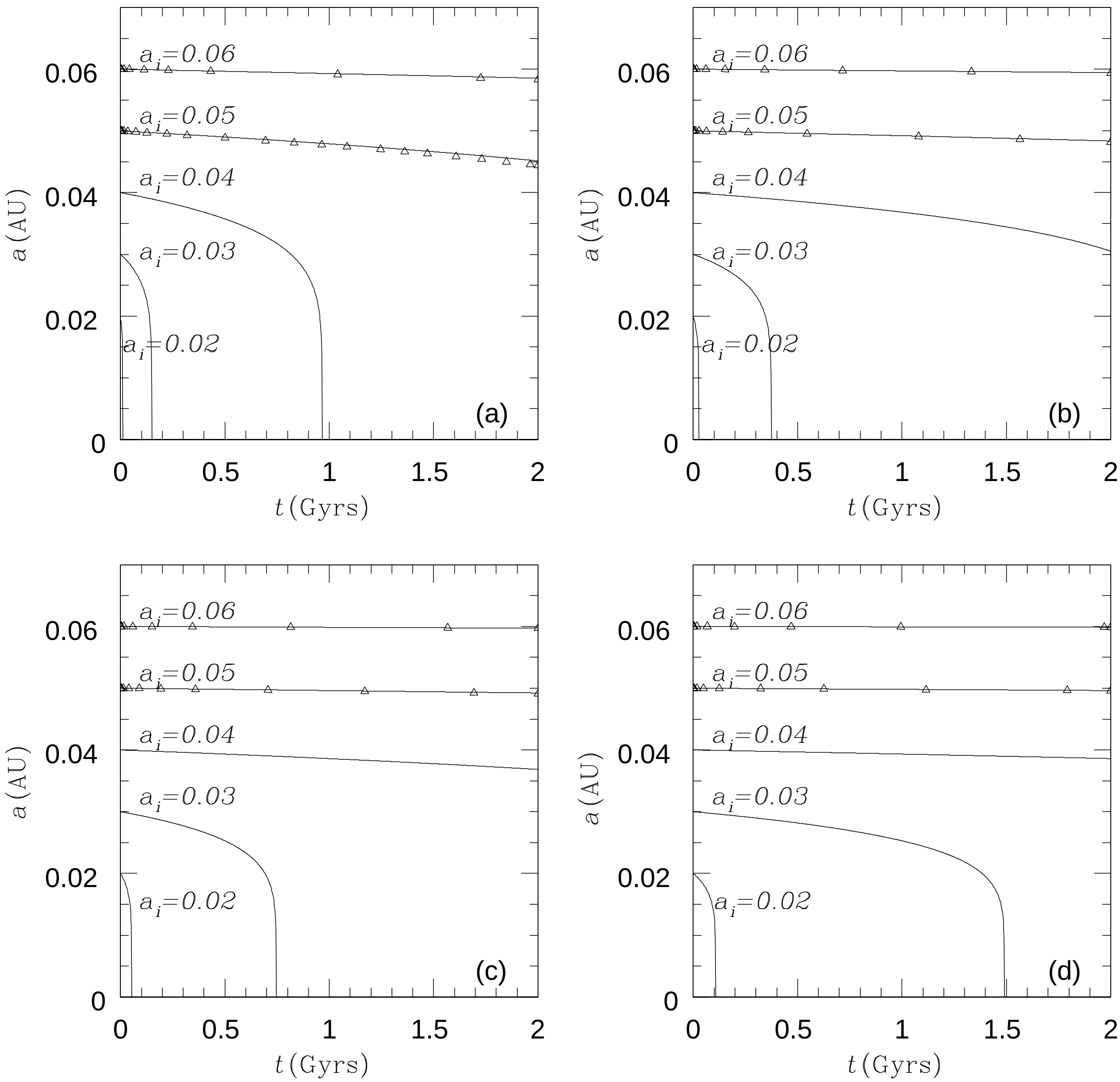

Fig. 1.— The semimajor axis as function of time. The solid curves are the results of $e = 0$ and initial semimajor axis $a_i = 0.02, 0.03, ..., 0.06$. The triangle points are the results of $e = 0.5$ and $a_i = 0.05, 0.06$. (a) Planetary mass is $5M_J$, (b) Planetary mass is $2M_J$, (c) Planetary mass is M_J, (d) Planetary mass is $0.5M_J$.

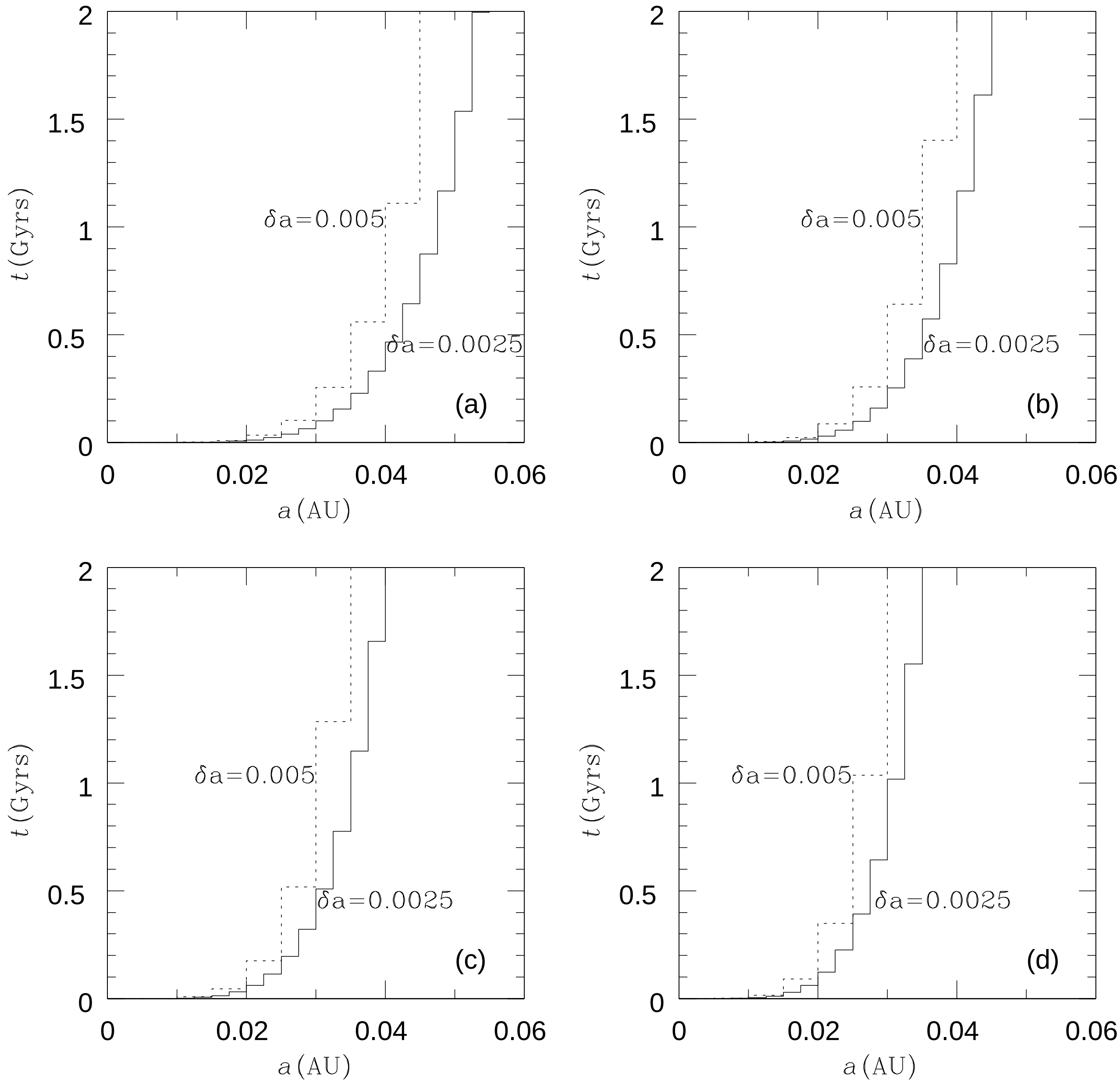

Fig. 2.— The migration time scales for give ranges of semimajor axis. The dotted lines are the results when $\delta a = 0.005$ and the solid lines are the results when $\delta a = 0.0025$. (a) Planetary mass is $5M_J$, (b) Planetary mass is $2M_J$, (c) Planetary mass is M_J, (d) Planetary mass is $0.5M_J$.

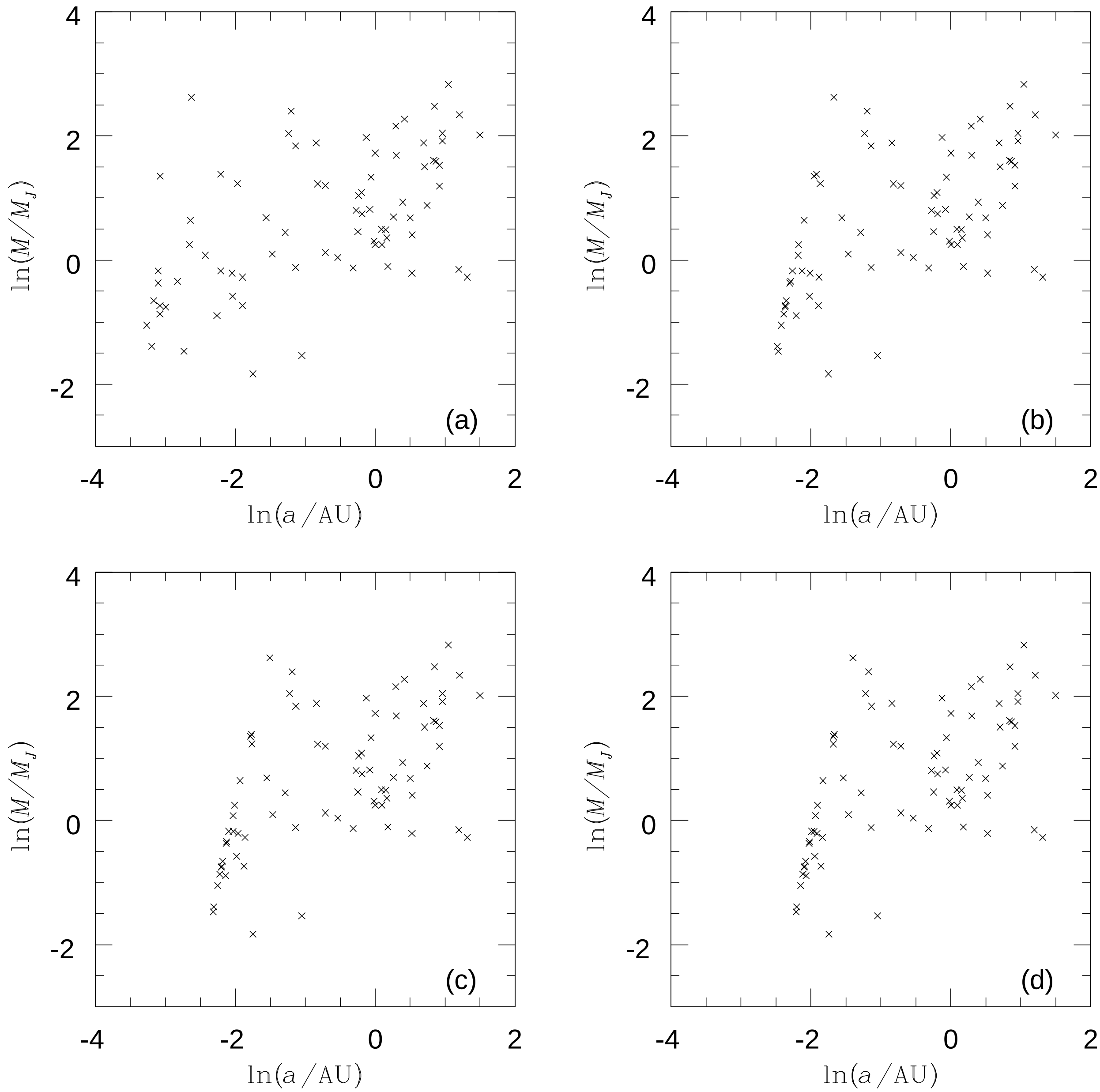

Fig. 3.— The $\ln(a/\mathrm{AU}) - \ln(M/M_J)$ plot for all discovered extrasolar planets in Extarsolar Planets Catalog in May 2002. (a) Current discovered configuration, (b) Backward in time for 2 Gyrs, (c) Backward in time for 6 Gyrs, (d) Backward in time for 12 Gyrs.